

A lattice NRQCD computation of the bag parameters for $\Delta B = 2$ operators*

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We present an update of our NRQCD calculation of B_B at $\beta=5.9$ with increased statistics. We also discuss a calculation of B_S , which is relevant to the width difference in the $B_s - \bar{B}_s$ mixing.

1. Introduction

The NRQCD calculation is essential to obtain a prediction for B_B with precision better than $O(20\%)$, as the size of $1/M$ correction, which is not included in the static calculations, is expected to be $\Lambda_{QCD}/m_b = 0.1 \sim 0.2$. We update our study of B_B using the NRQCD, which was previously presented at the last lattice conference [1]. A paper version is also available [2].

Based on the same calculation method, we have also calculated B_S , which is relevant to the width difference of B_s meson system [3].

2. Method

The bag parameter $B_{X_q}(m_b)$ is defined using a vacuum saturation approximation (VSA) as

$$B_{X_q}(m_b) = \frac{\langle \bar{B}_q^0 | \mathcal{O}_{X_q}(m_b) | B_q^0 \rangle}{\langle \bar{B}_q^0 | \mathcal{O}_{X_q}(m_b) | B_q^0 \rangle_{\text{VSA}}}, \quad (1)$$

where the $\Delta B=2$ operators \mathcal{O}_{X_q} are $\mathcal{O}_{L_q} = \bar{b}\gamma_\mu P_L q \bar{b}\gamma_\mu P_L q$ or $\mathcal{O}_{S_q} = \bar{b}P_L q \bar{b}P_L q$. (P_L is a projection operator $P_L = 1 - \gamma_5$.) Subscript q denotes the valence light quark d or s , which we omit in the following if there is no risk of confusion. We use a notation B_L instead of B_B to remind that it is a matrix element of \mathcal{O}_L and to distinguish it from \mathcal{O}_S .

Using the operators constructed on the lattice with static heavy and clover light quark $\mathcal{O}_X^{\text{lat}}$, the continuum operators defined with the \overline{MS} scheme

\mathcal{O}_X are written as

$$\mathcal{O}_L(m_b) = \sum_{X=\{L,S,R,N\}} Z_{L,X} \mathcal{O}_X^{\text{lat}}(a^{-1}), \quad (2)$$

$$\mathcal{O}_S(m_b) = \sum_{X=\{S,L,R,P\}} Z_{S,X} \mathcal{O}_X^{\text{lat}}(a^{-1}), \quad (3)$$

$$\mathcal{A}_0 = Z_A A_0^{\text{lat}}, \quad (4)$$

where new operators \mathcal{O}_R , \mathcal{O}_N and \mathcal{O}_P are involved:

$$\mathcal{O}_R = \bar{b}\gamma_\mu P_R q \bar{b}\gamma_\mu P_R q,$$

$$\mathcal{O}_N = 2 \bar{b}\gamma_\mu P_L q \bar{b}\gamma_\mu P_R q + 4 \bar{b}P_L q \bar{b}P_R q,$$

$$\mathcal{O}_P = 2 \bar{b}\gamma_\mu P_L q \bar{b}\gamma_\mu P_R q + 12 \bar{b}P_L q \bar{b}P_R q.$$

$Z_{L,X}$ and $Z_{S,X}$ are perturbative matching factors obtained at one-loop level [4]. We also write the matching of the heavy-light axial current \mathcal{A} with the renormalization constant Z_A .

The bag parameters are, then, written in terms of the corresponding quantities measured on the lattice B_X^{lat} as

$$B_L(m_b) = \sum_{X=\{L,S,R,N\}} Z_{L,X/A^2} B_X^{\text{lat}}(a^{-1}), \quad (5)$$

$$B_S(m_b)/0.734 = \sum_{X=\{S,L,R,P\}} Z_{S,X/A^2} B_X^{\text{lat}}(a^{-1}). \quad (6)$$

Here $Z_{L,X/A^2}$ denotes a ratio of matching constants $Z_{L,X}/Z_A^2$, and B_X^{lat} is defined by

$$B_X^{\text{lat}}(a^{-1}) = \frac{\langle \bar{B}^0 | \mathcal{O}_X^{\text{lat}}(a^{-1}) | B^0 \rangle}{c \langle \bar{B}^0 | A_\mu^{\text{lat}} | 0 \rangle \langle 0 | A_\mu^{\text{lat}} | B^0 \rangle}. \quad (7)$$

A numerical constant c is $8/3$ or $-5/3$ in B_L or in B_S respectively.

*Presented by N. Yamada.

q^*	$Z_{L,L/A^2}$	$Z_{L,S/A^2}$	$Z_{L,R/A^2}$	$Z_{L,N/A^2}$
π/a	0.973	-0.104	-0.007	-0.080
$1/a$	0.956	-0.172	-0.011	-0.132
q^*	$Z_{S,S/A^2}$	$Z_{S,L/A^2}$	$Z_{S,R/A^2}$	$Z_{S,P/A^2}$
π/a	1.307	0.032	0.002	0.010
$1/a$	1.505	0.053	0.003	0.017

Table 1
Perturbative matching factors at $\beta=5.9$.

The vacuum saturation of the operator \mathcal{O}_S introduces a matrix element of the pseudoscalar density $P = \bar{b}\gamma_5 q$, which is often rewritten in terms of A_μ using the equation of motion. In doing so, a factor $(\bar{m}_b(m_b) + \bar{m}_s(m_b))^2/M_{B_s}^2$ appears, for which we use

$$\begin{aligned} m_b &= 4.8 \text{ GeV}, & \bar{m}_b(m_b) &= 4.4 \text{ GeV}, \\ \bar{m}_s(m_b) &= 0.2 \text{ GeV}, & M_{B_s} &= 5.37 \text{ GeV} \end{aligned}$$

as in Ref.[3], and obtain 0.734 given in Eq.(6).

Unfortunately the one-loop coefficients for the perturbative matching are not yet available for the NRQCD action. We use, therefore, the one-loop coefficients calculated in the static limit as an approximation. It introduces a systematic error of $O(\alpha_s/(am_Q))$, but no logarithmic divergence appears. The numerical values of $Z_{L,X/A^2}$ and $Z_{S,X/A^2}$ at $\beta=5.9$ are given in Table 2, in which we linearize the perturbative expansion of $Z_{L,X}/Z_A^2$ and neglect all the $O(\alpha_s^2)$ terms. For the coupling constant, $\alpha_V(q^*)$ with $q^* = 1/a$ and π/a is used throughout this paper.

Our simulation was carried out on a quenched $16^3 \times 48$ lattice at $\beta=5.9$. We have increased the statistics to 250 from 100 at the time of Lattice 98 [1]. We performed two sets of simulations with the NRQCD actions and currents improved through $O(1/m_Q)$ and $O(1/m_Q^2)$, which enables us to study the higher order effects in the $1/m_Q$ expansion explicitly. The light quark is described by the clover action with the tadpole improved clover coefficient $c_{sw} = 1/u_0^3$. The inverse lattice spacing is determined from the rho meson mass as $a^{-1} = 1.62$ GeV.

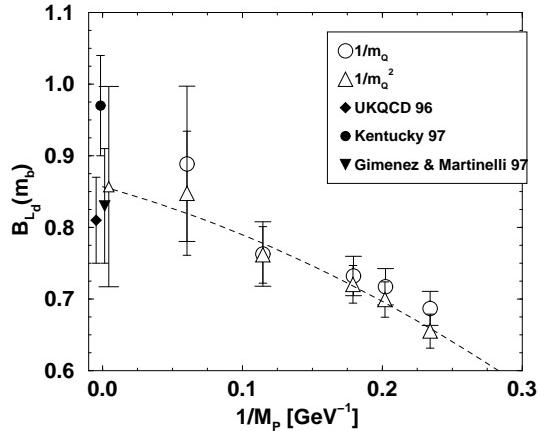


Figure 1. The heavy quark mass dependence of $B_{L_d}(m_b)$.

3. B_L

Figure 1 shows $1/M_P$ dependence of $B_{L_d}(m_b)$ ($q^*=1/a$) with M_P the pseudoscalar heavy-light meson mass. Open circles denote the results with the $O(1/m_Q)$ NRQCD action and open triangles denote those with the $O(1/m_Q^2)$ action.

We fit the $O(1/m_Q^2)$ results to a quadratic function of $1/M_P$ (dashed line) and obtain the value in the static limit (small open triangle). We also plot the previous results in the static limit by UKQCD [5] (filled diamond), Kentucky group [6] (filled circle) and Giménez and Martinelli [7] (filled triangle). In order to make a consistent comparison we reanalyzed their data using the same matching procedure described in the last section. Our data extrapolated to the static limit nicely agrees with these direct simulation results, as it should be.

From Figure 1 we observe that B_L has a small negative slope in $1/M_P$, which is well described by the vacuum saturation approximation [1,2] and also observed in the lattice calculations with relativistic actions [8,9]. We also find that the $O(1/m_Q^2)$ corrections to the action and current gives only a few per cent contribution to B_L .

The dominant uncertainty in our result comes from the unknown one-loop coefficients for the NRQCD action. A crude estimate with order counting suggests that the corresponding systematic error is $O(\alpha_s/(am_b)) \sim 10\%$. Other possible

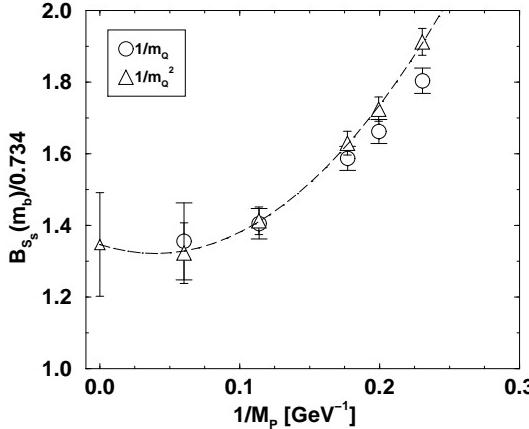


Figure 2. The heavy quark mass dependence of $B_{S_s}(m_b)$.

systematic errors are the discretization error of $O(a^2 \Lambda_{QCD}^2)$ and of $O(\alpha_s a \Lambda_{QCD})$, the relativistic correction of $O(\Lambda_{QCD}^2 / m_b^2)$, and a small uncertainty in the chiral extrapolation.

Taking them into account, we obtain the following values as our final results from the quenched lattice,

$$B_{B_d}(m_b) = 0.75(3)(12), \quad \frac{B_{B_s}}{B_{B_d}} = 1.01(1)(3), \quad (8)$$

where the first error is statistical and the second a sum of the systematic errors in quadrature. In estimating the error in the ratio B_{B_s}/B_{B_d} we consider the error from chiral extrapolation only, assuming that other uncertainties cancel in the ratio.

4. B_S

Figure 2 shows the $1/M_P$ dependence of $B_{S_s}(m_b)$ with $q^* = 1/a$. We see a significant increase of B_S with the $1/M$ correction, which is 20~30%. Our preliminary result with a similar error analysis as in B_L is

$$B_{S_s}(m_b) = 1.19(2)(20). \quad (9)$$

The width difference in the $B_s - \bar{B}_s$ mixing $\Delta\Gamma_s$

is theoretically calculated using the $1/M$ expansion as [3]

$$\left(\frac{\Delta\Gamma}{\Gamma} \right)_s = \left(\frac{f_{B_s}}{210 \text{ MeV}} \right)^2 \times [0.006 B_{L_s}(m_b) + 0.150 B_{S_s}(m_b) - 0.063].$$

Using our result for B_L and B_S , and a recent dynamical lattice result $f_{B_s} = 245(30)$ MeV [9], we obtain $(\Delta\Gamma/\Gamma)_s = 0.16(3)(4)$, where errors are from f_{B_s} and from B_S respectively.

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